Results on finite-time teleoperation of robotic systems

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Abstract. A bilateral teleoperation system comprises a network of two manipulators, with the primary objective of achieving position synchronization between them. This paper proposes a novel continuous-time finite-time (FT) controller to solve the problem of bilateral teleoperation for a class of fully-actuated Euler-Lagrange (EL) systems in joint space. For this, the control law is designed to ensure global asymptotic stability (GAS) and then, finite-time stability (FTS) through a homogeneous approximation of negative degree. To handle the external forces that arise from environmental interactions, it is assumed that these forces define passive velocity-to-force maps, enabling the proof of bounded position errors and velocities. The resulting controller is an easy-to-implement proportional plus damping injection (P+d) scheme with a gravity cancellation term. Experimental results with a teleoperation system composed of two robotic manipulators with six Degrees-of-Freedom (DoF) are presented to validate the performance of the proposed scheme.

Keywords: Finite-time control · Euler-Lagrange systems · Robotics.

1 Introduction

A bilateral teleoperation system consists of the interconnection of a local and a remote manipulator, where the interaction of external forces should be considered [3]. As it is typically described, a human operator interacts with the local manipulator to change its position, then, this interaction is transmitted to the remote manipulator in order to mimic the movements of the local. The result of this collaboration, is to extend the capabilities of the operator to control the behavior of a remote manipulator in order to execute task in another location, which is considered prone to environmental disturbances.

To this day, teleoperation system have been of particular interest in engineering due to its large amount of applications [10, 15], with notable success in the medical industry [7]. As a result, numerous studies have addressed the teleoperation problem. However, most existing solutions only guarantee asymptotic stability (AS) [1, 9, 11].

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This work focuses on controllers that achieve finite-time stability (FTS). In addition to their convergence properties, finite-time controllers offer faster transient responses, improved precision, and enhanced robustness compared to conventional asymptotic controllers [5]; These advantages make finite-time controllers particularly appealing for teleoperation systems, motivating the development of novel control strategies to leverage these benefits.

Current FT solutions are mostly based in discontinuous controllers, in particular, the sliding-mode technique: among these works it is possible to find controllers that only ensure FT convergence to a vicinity or the origin [6, 16], while solutions that achieve global FT convergence are not that common [14, 17]; in any case, the main disadvantage of these controllers is the chattering effect that ends to deteriorate the systems actuator due to its aggressive control law [12]. Solutions using a continuous-time control law are found in [8, 18], but again, they only ensure FT convergence to a vicinity of the origin.

This paper presents a continuous-time control solution that ensures global finite-time convergence. The proposed controller combines a proportional term for position error, a damping injection term, and gravity compensation to cancel gravitational effects. Under the assumption that the remote environment and the human operator define passive velocity-to-force maps, it is demonstrated that while external forces act on the system, the position error between the local and remote manipulators, as well as their velocities, remain bounded. On the other hand, in the absence of external forces, both the position error and the velocities converge to zero in finite-time. To test the performance of the controller, it is presented experimental results from a teleoperation system composed of two Kinova® robots with six DoF, furthermore, it is also presented a comparison against a similar asymptotic scheme.

2 Foundations

This article uses the following *notation*:

- $\mathbb{R} := (-\infty, \infty)$; $\mathbb{R}_{\geq 0} := [0, \infty)$; $\mathbb{N} := \{1, 2, 3, \dots\}$; and $\bar{n} := \{1, 2, ..., n\}$ for all $n \in \mathbb{N}$.
- For $x \in \mathbb{R}$, $|x|$ is its absolute value. For $\mathbf{x} \in \mathbb{R}^m : m \in \mathbb{N}$, $\|\mathbf{x}\|$ refers to the Euclidean norm.
- For all $\delta \in \mathbb{R}_{>0}$, $B_\delta := \{\mathbf{x} \in \mathbb{R}^m : \|\mathbf{x}\| < \delta\}$ and $S_\delta^{m-1} := \{\mathbf{x} \in \mathbb{R}^m : \|\mathbf{x}\| = \delta\}$, are an open ball and a sphere of dimension $m-1$, respectively, with their center in the origin and a radius of δ.
- A function $\mathbf{f}(t) : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^m$ is said to be class $\mathcal{C}^k$, for all $k \in \mathbb{N}$, if its derivatives $\dot{\mathbf{f}}, \ddot{\mathbf{f}}(\mathbf{t}), ..., \mathbf{f}^{(k)}$ exist and are continuous.

For all $x \in \mathbb{R}$ and $p \in \mathbb{R}_{>0}$, we define the function *signed power* $\lceil x \rfloor^p : \mathbb{R} \mapsto \mathbb{R}$ as an odd strictly increasing function $\lceil x \rfloor^p := |x|^p \mathrm{sign}\,(x)$, where $\mathrm{sign}\,(x)$ is the standard *sign function*. The *signed power* function have the next properties:

Property 1. For each $p \in [0, 1)$, $\lceil x \rfloor^p$ is differentiable $\forall x \neq 0$, $\lceil x \rfloor^p \in \mathcal{C}^1$ if $p \in [1, 2]$ and $\lceil x \rfloor^p \in \mathcal{C}^2$ if $p \in (2, \infty)$.

Property 2. $\partial_x \lceil x \rfloor^p = p|x|^{p-1}$ and $\partial_x |x|^p = p\lceil x \rfloor^{p-1}$, $\forall x \neq 0$.

2.1 Finite-time Stability

Consider a dynamic system described by

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \quad \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^m$ is the state vector, $\mathbf{f} : \mathbb{R}^m \mapsto \mathbb{R}^m$ is the continuous vector field asociated, and $m \in \mathbb{N}$. Assume the origin is the equilibrium point, i.e. $\mathbf{f}(\mathbf{0}) = \mathbf{0}$.

Definition 1. *The origin of (1) is finite-time stable if it is Lyapunov stable and if exists a local-bounded function* $T : B_\delta \mapsto \mathbb{R}_{\geq 0}$ *(called the settling-time function) such that for each* $\mathbf{x}_0 \in B_\delta \setminus \{\mathbf{0}\}$*, every solution* $\mathbf{x}(t, \mathbf{x}_0)$ *of (1) is defined in* $t \in [0, T(\mathbf{x}_0))$ *and* $\mathbf{x}(t, \mathbf{x}_0) = \mathbf{0}$ *for all* $t \geq T(\mathbf{x}_0)$*. If* $B_\delta = \mathbb{R}^m$, $\mathbf{x} = \mathbf{0}$ *is globally FTS [4, 5].*

Definition 2. *Let's define* $r_i > 0$, $i \in \bar{m}$ *as the weights of the elements* x_i *of* $\mathbf{x} \in \mathbb{R}^m$*, and the vector of weights* $\boldsymbol{r} := [r_1, ..., r_m]^\top \in \mathbb{R}^m$*. Also, it is defined the dilation operator* $\Delta_\epsilon^{\mathbf{r}}$*, such that* $\Delta_\epsilon^{\mathbf{r}}\mathbf{x} := [\epsilon^{r_1}x_1, ..., \epsilon^{r_m}x_m]^\top$*. A function* $V : \mathbb{R}^m \mapsto \mathbb{R}$ *(resp. a vector field* $\mathbf{f} : \mathbb{R}^m \mapsto \mathbb{R}^m$*) is* $\mathbf{r}$*-homogeneous of degree* $l \in \mathbb{R}$*, or* $(\mathbf{r}, l)$*-homogeneous, if for all* $\epsilon \in \mathbb{R}_{>0}$ *and* $\mathbf{x} \in \mathbb{R}^m$ *satisfies the equality* $V(\Delta_\epsilon^{\mathbf{r}}\mathbf{x}) = \epsilon^l V(\mathbf{x})$ *(resp.,* $\mathbf{f}(\Delta_\epsilon^{\mathbf{r}}\mathbf{x}) = \epsilon^l \Delta_\epsilon^{\mathbf{r}}\mathbf{f}(\mathbf{x})$*). The system (1) is called* $(\mathbf{r}, l)$*-homogeneous if the vector field* $\mathbf{f}$ *is* $(\mathbf{r}, l)$*-homogeneous [2].*

Lemma 1. *Consider the system (1) with* $\mathbf{f}(\mathbf{x}) = \mathbf{f}_H(\mathbf{x}) + \mathbf{f}_{NH}(\mathbf{x})$*. Suppose* $\mathbf{f}_H(\mathbf{x})$ *is a continuous vector field* $(\mathbf{r}, l)$*-homogeneous such that* $\mathbf{f}_H(\mathbf{0}) = \mathbf{0}$ *is a locally asymptotically stable (AS) equilibrium point of* $\dot{\mathbf{x}} = \mathbf{f}_H(\mathbf{x})$*. Assume* $\mathbf{f}_{NH}(\mathbf{x})$ *is a continuous vector field that* $\mathbf{f}_{NH}(\mathbf{0}) = \mathbf{0}$ *and the next vanishing condition*

$$\lim_{\epsilon \to 0} \epsilon^{-(l+r_i)} f_{NH_i}(\Delta_\epsilon^{\mathbf{r}}\mathbf{x}) = 0$$

is uniformly satisfied with respect to $\mathbf{x} \in S_\delta^{m-1}$ *for* $\delta \in \mathbb{R}_{>0}$ *and all* $i \in \bar{m}$*. Then, the origin of (1) is locally AS. Also, if* $l = 0$ *and every* $r_i = 1$*, the origin is locally and exponentially stable (ES); and if* $l < 0$*, the origin is FTS [2].*

Lemma 2. *Assume that the system (1) admits an homogeneous approximation of the form* $\dot{\mathbf{x}} = \mathbf{f}_H(\mathbf{x})$ *such that he vector field* $\mathbf{f}_H(\mathbf{x})$ *is* $(\mathbf{r}, l)$*-homogeneous and* $\mathbf{x} = \mathbf{0}$ *is AS. Also, if* $\mathbf{x} = \mathbf{0}$ *is globally AS for system (1), then, the origin of (1) is globally AS and locally ES if* $l = 0$ *y* $r_i = 1$*; and it is globally FTS if* $l < 0$*.*

Remark 1. In the stability proofs, the resulting closed-loop system can be written as (1), and always can be separated into homogeneous and not homogeneous terms. Then, $\mathbf{f}_H(\mathbf{x})$ always can be found in such a way that the system (1) is converted to $\dot{\mathbf{x}} = \mathbf{f}_H(\mathbf{x}) + \mathbf{f}_{NH}(\mathbf{x})$. In this case, $\mathbf{f}_{NH}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) - \mathbf{f}_H(\mathbf{x})$, and the *vanishing condition* is equivalent to $\lim_{\epsilon \to 0} \sup_{\mathbf{x} \in S_\delta^{m-1}} \|\epsilon^{-l}(\Delta_\epsilon^{\mathbf{r}})^{-1}\mathbf{f}(\Delta_\epsilon^{\mathbf{r}}\mathbf{x}) - \mathbf{f}_H(\mathbf{x})\| = 0$.

2.2 The Dynamic Model

The local and remote manipulators that constitute the teleoperation system are modeled as a set of rigid links connected in series with n fully-actuated revolution joints that allow the movement in n degrees-of-freedom. The Euler-Lagrange equations that describe the dynamic of each manipulator are:

$$\begin{aligned} \mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \nabla_{\mathbf{q}_l}\mathcal{U}_l(\mathbf{q}_l) &= \boldsymbol{\tau}_l + \boldsymbol{\tau}_h, \\ \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \nabla_{\mathbf{q}_r}\mathcal{U}_r(\mathbf{q}_r) &= \boldsymbol{\tau}_r + \boldsymbol{\tau}_e, \end{aligned} \tag{2}$$

where $\mathbf{q}_i, \dot{\mathbf{q}}_i$ and $\ddot{\mathbf{q}}_i \in \mathbb{R}^n$ with $i \in \{l, r\}$ are the joint position, velocity and acceleration, respectively. $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ the Coriolis and centrifugal forces matrix, defined by the Christoffel symbols of first kind. $\mathcal{U}_i$ is the potential energy for each manipulator, $\boldsymbol{\tau}_i \in \mathbb{R}^n$ are the control signals, $\boldsymbol{\tau}_h \in \mathbb{R}^n$ and $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the joints forces caused by the human operator and the environmental interactions, respectively. The previous model has the following properties:

Property 3. Exist strictly positive constants m_{1i} y m_{2i}, such that $m_{1i} \leq \mathbf{M}_i(\mathbf{q}_i) \leq m_{2i}$, for all $\mathbf{q}_i \in \mathbb{R}^n$ for $i \in \{l, r\}$.

Property 4. The matrix $\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is skew-symmetric and exist $L_{ci} > 0$ such that $\|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i\| \leq L_{ci}\|\dot{\mathbf{q}}_i\|^2$.

Also, we make the following assumption, which is ubiquitous in the control of teleoperation systems via passivity-based techniques [11].

Assumption 1. *The human operator and the environment define passive (velocity to force) maps, this is, there exist κ_l and $\kappa_r \in \mathbb{R}_{\geq 0}$, such that:*

$$\int_0^t \dot{\mathbf{q}}_l^\top(\sigma)\boldsymbol{\tau}_h(\sigma)d\sigma \leq \kappa_l, \qquad \int_0^t \dot{\mathbf{q}}_r^\top(\sigma)\boldsymbol{\tau}_e(\sigma)d\sigma \leq \kappa_r, \tag{3}$$

for all $t \geq 0$.

3 The Teleoperation System

3.1 Problem Statement

For a teleoperation system define as (2), the objective is to design a controller that ensures that the position error between manipulators and their corresponding velocities remain bounded, i.e., $\mathbf{q}_l - \mathbf{q}_r$, $\dot{\mathbf{q}}_l$, $\dot{\mathbf{q}}_r \in \mathcal{L}_\infty$. Then, when this forces vanishes ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = 0$), both manipulator must reach a constant consensus position, this is, there exist a constant $\mathbf{q}_* \in \mathbb{R}^n$, such that:

$$\lim_{t\to T_i(\mathbf{q}_i(0),\dot{\mathbf{q}}_i(0))} \mathbf{q}_i(t) = \mathbf{q}_*, \qquad \lim_{t\to T_i(\mathbf{q}_i(0),\dot{\mathbf{q}}_i(0))} |\dot{\mathbf{q}}_i(t)| = \mathbf{0}, \tag{4}$$

where $i := \{l, r\}$ and $T_i(\mathbf{q}_i(0), \dot{\mathbf{q}}_i(0)) < \infty$ is a settling-time function.

Considering this, the following control law is proposed:

$$\begin{aligned}\boldsymbol{\tau}_l &= -K_l\lceil \mathbf{q}_l - \mathbf{q}_r \rfloor^{p_{\mathcal{U}}} - B_l\lceil \dot{\mathbf{q}}_l \rfloor^{p_{\mathcal{F}}} + \nabla_{\mathbf{q}_l}\mathcal{U}_l(\mathbf{q}_l),\\ \boldsymbol{\tau}_r &= -K_r\lceil \mathbf{q}_r - \mathbf{q}_l \rfloor^{p_{\mathcal{U}}} - B_r\lceil \dot{\mathbf{q}}_r \rfloor^{p_{\mathcal{F}}} + \nabla_{\mathbf{q}_r}\mathcal{U}_r(\mathbf{q}_r),\end{aligned} \tag{5}$$

where $K_l,\, K_r \in \mathbb{R}_{>0}$ are the proportional gains; $B_l,\, B_r \in \mathbb{R}_{>0}$ are the damping injection gains; and the coefficients $p_{\mathcal{U}},\, p_{\mathcal{F}} \in (0,1)$ are the key to guarantee FT stability, and are defined as:

$$p_{\mathcal{U}} = \frac{2r_2 - r_1}{r_1}, \qquad p_{\mathcal{F}} = \frac{2r_2 - r_1}{r_2}, \tag{6}$$

with $r_1,\, r_2 \in \mathbb{R}_{>0}$ satisfying:

$$2r_2 > r_1 > r_2 > 0. \tag{7}$$

With this, the control law (5) solves the finite-time teleoperation problem. Before presenting the proof of this claim, it is important to notice that showing global FT convergence to (4) is equivalent to prove global FTS to:

$$\left(\mathbf{q}_l - \mathbf{q}_r,\, \dot{\mathbf{q}}_l,\, \dot{\mathbf{q}}_r\right) = \left(\mathbf{0},\, \mathbf{0},\, \mathbf{0}\right). \tag{8}$$

Now, we are ready to present the main result of this work.

Proposition 1. *Consider the bilateral teleoperated system (2), set the control law (5) with $\boldsymbol{\tau}_h$ and $\boldsymbol{\tau}_e$ meeting Assumption 1, choose $p_{\mathcal{U}}$ and $p_{\mathcal{F}}$ as in (6) and (7), then:*

A. *Position error and velocities remain bounded $\mathbf{q}_l - \mathbf{q}_r,\, \dot{\mathbf{q}}_l,\, \dot{\mathbf{q}}_r \in \mathcal{L}_\infty$.*
B. *In addition, if $\boldsymbol{\tau}_h = 0$ and $\boldsymbol{\tau}_e = 0$, then, the equilibrium point (8) is globally FTS.*

Proof. Consider the following non-negative function $\mathcal{V}(t)$, which is in function of the coordinates of the equilibrium point (8):

$$\begin{aligned}\mathcal{V}(t) = {} & \frac{1}{2}\dot{\mathbf{q}}_l^\top \mathbf{M}_l \dot{\mathbf{q}}_l + \frac{K_l}{2K_r}\dot{\mathbf{q}}_r^\top \mathbf{M}_r \dot{\mathbf{q}}_r + \frac{K_l}{p_{\mathcal{U}}+1}(\mathbf{q}_r - \mathbf{q}_l)^\top \lceil \mathbf{q}_l - \mathbf{q}_r \rfloor^{p_{\mathcal{U}}}\\ & + \kappa_l + \frac{K_l}{K_r}\kappa_r - \int_0^t \left(\dot{\mathbf{q}}_l^\top(\sigma)\boldsymbol{\tau}_h(\sigma) + \frac{K_l}{K_r}\dot{\mathbf{q}}_r^\top(\sigma)\boldsymbol{\tau}_e(\sigma)\right) d\sigma.\end{aligned} \tag{9}$$

Then, use Property 4 and evaluate (9) along the system trajectories (2):

$$\dot{\mathcal{V}}(t) = -B_l\dot{\mathbf{q}}_l^\top \lceil \dot{\mathbf{q}}_l \rfloor^{p_{\mathcal{F}}} - \frac{K_l}{K_r}B_r\dot{\mathbf{q}}_r^\top \lceil \dot{\mathbf{q}}_r \rfloor^{p_{\mathcal{F}}} \le 0. \tag{10}$$

Now, time-integrate $\dot{\mathcal{V}}(t)$ from 0 to t and get:

$$\mathcal{V}(t) - \mathcal{V}(0) = -\int_0^t \left(B_l\dot{\mathbf{q}}_l^\top \lceil \dot{\mathbf{q}}_l \rfloor^{p_{\mathcal{F}}} + \frac{K_l}{K_r}B_r\dot{\mathbf{q}}_r^\top \lceil \dot{\mathbf{q}}_r \rfloor^{p_{\mathcal{F}}}\right) d\sigma, \tag{11}$$

and notice that the term inside the integral in (11) is always non-negative, making possible to find an upper-bound for $\mathcal{V}(t)$ as:

$$\mathcal{V}(t) \le \mathcal{V}(0) < \infty,$$

this implies that vectors $\mathbf{q}_l - \mathbf{q}_r$, $\dot{\mathbf{q}}_l$, $\dot{\mathbf{q}}_r \in \mathcal{L}_\infty$, proving conclusion A of Proposition 1.

Now, we consider the case of no external forces, for this, take equation (9) with $\boldsymbol{\tau}_h = 0$ and $\boldsymbol{\tau}_e = 0$, and also consider $\kappa_l = 0$ and $\kappa_r = 0$:

$$\mathcal{V}(t) = \frac{1}{2}\dot{\mathbf{q}}_l^\top \mathbf{M}_l \dot{\mathbf{q}}_l + \frac{K_l}{2K_r}\dot{\mathbf{q}}_r^\top \mathbf{M}_r \dot{\mathbf{q}}_r + \frac{K_l}{p_{\mathcal{U}}+1}(\mathbf{q}_r - \mathbf{q}_l)^\top \lceil \mathbf{q}_l - \mathbf{q}_r \rfloor^{p_{\mathcal{U}}}, \tag{12}$$

as before, taking the time-derivative of $\mathcal{V}(t)$ leads to the same expression (10). Then, using LaSalle's Invariance Principle we can conclude global AS of (8).

To achieve global FTS it is necessary to prove that the teleoperation system along with the controller admits and homogeneous approximation of negative degree. First, we will express the closed-loop system in a state-space representation $\Sigma : \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ as follows:

$$\Sigma := \begin{cases} \dot{\tilde{\mathbf{q}}}_l = \dot{\mathbf{q}}_l \\ \dot{\tilde{\mathbf{q}}}_r = \dot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_l = -\mathbf{M}_l^{-1}(\tilde{\mathbf{q}}_l + \mathbf{q}_*)\Big[K_l\lceil \tilde{\mathbf{q}}_l - \tilde{\mathbf{q}}_r \rfloor^{p_{\mathcal{U}}} + B_l \lceil \dot{\mathbf{q}}_l \rfloor^{p_{\mathcal{F}}} + \mathbf{C}_l(\tilde{\mathbf{q}}_l + \mathbf{q}_*, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l\Big] \\ \ddot{\mathbf{q}}_r = -\mathbf{M}_r^{-1}(\tilde{\mathbf{q}}_r + \mathbf{q}_*)\Big[K_r\lceil \tilde{\mathbf{q}}_r - \tilde{\mathbf{q}}_l \rfloor^{p_{\mathcal{U}}} + B_r \lceil \dot{\mathbf{q}}_r \rfloor^{p_{\mathcal{F}}} + \mathbf{C}_r(\tilde{\mathbf{q}}_r + \mathbf{q}_*, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r\Big] \end{cases} \tag{13}$$

where $\tilde{\mathbf{q}}_l = \mathbf{q}_l - \mathbf{q}_*$ y $\tilde{\mathbf{q}}_r = \mathbf{q}_r - \mathbf{q}_*$ are the errors between the joint position of each manipulator with respect to the consensus position.

Now, we split $\mathbf{f}(\mathbf{x})$ into homogeneous and non-homogeneous terms, such that $\mathbf{f}(\mathbf{x}) = \mathbf{f}_H(\mathbf{x}) + \mathbf{f}_{NH}(\mathbf{x})$, where $\mathbf{f}_H(\mathbf{0}) = \mathbf{0}$ must be an AS equilibrium point for system $\Sigma_H : \dot{\mathbf{x}} = \mathbf{f}_H(\mathbf{x})$ and be homogeneous of negative degree. So, the homogeneous system Σ_H is selected this way:

$$\Sigma_H := \begin{cases} \dot{\tilde{\mathbf{q}}}_l = \dot{\mathbf{q}}_l \\ \dot{\tilde{\mathbf{q}}}_r = \dot{\mathbf{q}}_r \\ \ddot{\mathbf{q}}_l = -\mathbf{M}_l^{-1}(\mathbf{q}_*)\Big[K_l\lceil \tilde{\mathbf{q}}_l - \tilde{\mathbf{q}}_r \rfloor^{p_{\mathcal{U}}} + B_l \lceil \dot{\mathbf{q}}_l \rfloor^{p_{\mathcal{F}}}\Big] \\ \ddot{\mathbf{q}}_r = -\mathbf{M}_r^{-1}(\mathbf{q}_*)\Big[K_r\lceil \tilde{\mathbf{q}}_r - \tilde{\mathbf{q}}_l \rfloor^{p_{\mathcal{U}}} + B_r \lceil \dot{\mathbf{q}}_r \rfloor^{p_{\mathcal{F}}}\Big] \end{cases} \tag{14}$$

and the homogeneity weights $\mathbf{r} = \begin{bmatrix} r_1 \; r_1 \; r_2 \; r_2 \end{bmatrix}$ are asigned to the vector field. This choice gives that Σ_H is $(\mathbf{r}, r_2 - r_1)$-homogeneous and, if r_1 and r_2 meet (7), then it is homogeneous of negative degree.

The next step is to prove that $\mathbf{f}_H(\mathbf{0}) = \mathbf{0}$ is AS, for this, consider the following function, which is the homogeneous version of (9):

$$\mathcal{V}_H = \frac{1}{2}\dot{\mathbf{q}}_l^\top \mathbf{M}_l(\mathbf{q}_*) \dot{\mathbf{q}}_l + \frac{K_l}{2K_r}\dot{\mathbf{q}}_r^\top \mathbf{M}_r(\mathbf{q}_*) \dot{\mathbf{q}}_r + \frac{K_l}{p_{\mathcal{U}}+1}(\mathbf{q}_r - \mathbf{q}_l)^\top \lceil \mathbf{q}_l - \mathbf{q}_r \rfloor^{p_{\mathcal{U}}} \tag{15}$$

and its time-derivative gives:

$$\dot{\mathcal{V}}_H = -B_l \dot{\mathbf{q}}_l^\top \lceil \dot{\mathbf{q}}_l \rfloor^{p_{\mathcal{F}}} - \frac{K_l}{K_r} B_r \dot{\mathbf{q}}_r^\top \lceil \dot{\mathbf{q}}_r \rfloor^{p_{\mathcal{F}}} \leq 0 \tag{16}$$

and using LaSalle's Invariance Principle it is proved that (8) is AS for Σ_H.

The last step is to prove that Σ_H is a valid homogeneous approximation of system Σ. For this, it is requiered that the non-homogeneous terms $\mathbf{f}_{NH}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) - \mathbf{f}_H(\mathbf{x})$ meet the *vanishing conditions*. To start, notice that the first elements $\mathbf{f}_{NH1} = 0$ and $\mathbf{f}_{NH2} = 0$ already satisfy this conditions. Then, using algebra manipulations we can obtain the following expression for $\mathbf{f}_{NH3}$:

$$\begin{aligned}\mathbf{f}_{NH3} = &- \left[\mathbf{M}_l^{-1}(\tilde{\mathbf{q}}_l + \mathbf{q}_*) - \mathbf{M}_l^{-1}(\mathbf{q}_*)\right] \left[K_l \lceil \tilde{\mathbf{q}}_l - \tilde{\mathbf{q}}_r \rfloor^{p_\mathcal{U}} + B_l \lceil \dot{\mathbf{q}}_l \rfloor^{p_\mathcal{F}}\right] \\ &- \mathbf{M}_l^{-1}(\tilde{\mathbf{q}}_l + \mathbf{q}_*)\mathbf{C}_l(\tilde{\mathbf{q}}_l + \mathbf{q}_*, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l,\end{aligned}$$

To prove that $\mathbf{f}_{NH3}$ meets the vanishing conditions, we will prove that each element inside it also meets them, starting with the first element:

$$\lim_{\epsilon \to 0} \frac{\left\| \left[\mathbf{M}_l^{-1}(\epsilon^{r_1}\tilde{\mathbf{q}}_l + \mathbf{q}_*) - \mathbf{M}_l^{-1}(\mathbf{q}_*)\right] \left[K_l \lceil \epsilon^{r_1}\tilde{\mathbf{q}}_l - \epsilon^{r_1}\tilde{\mathbf{q}}_r \rfloor^{p_\mathcal{U}} + B_l \lceil \epsilon^{r_2}\dot{\mathbf{q}}_l \rfloor^{p_\mathcal{F}}\right] \right\|}{\epsilon^{2r_2 - r_1}},$$

here, notice that $K_l \lceil \epsilon^{r_1}\tilde{\mathbf{q}}_l - \epsilon^{r_1}\tilde{\mathbf{q}}_r \rfloor^{p_\mathcal{U}} = \epsilon^{r_1 p_\mathcal{U}} K_l \lceil \tilde{\mathbf{q}}_l - \tilde{\mathbf{q}}_r \rfloor^{p_\mathcal{U}}$ and $B_l \lceil \epsilon^{r_2}\dot{\mathbf{q}}_l \rfloor^{p_\mathcal{F}} = \epsilon^{r_2 p_\mathcal{F}} B_l \lceil \dot{\mathbf{q}}_l \rfloor^{p_\mathcal{F}}$ and, if (6) holds, we get:

$$\begin{aligned}&\lim_{\epsilon \to 0} \left\| \left[\mathbf{M}_l^{-1}(\epsilon^{r_1}\tilde{\mathbf{q}}_l + \mathbf{q}_*) - \mathbf{M}_l^{-1}(\mathbf{q}_*)\right] \left[K_l \lceil \tilde{\mathbf{q}}_l - \tilde{\mathbf{q}}_r \rfloor^{p_\mathcal{U}} + B_l \lceil \dot{\mathbf{q}}_l \rfloor^{p_\mathcal{F}}\right] \right\| \\ &\quad = \left\| \left[\mathbf{M}_l^{-1}(\mathbf{q}_*) - \mathbf{M}_l^{-1}(\mathbf{q}_*)\right] \left[K_l \lceil \tilde{\mathbf{q}}_l - \tilde{\mathbf{q}}_r \rfloor^{p_\mathcal{U}} + B_l \lceil \dot{\mathbf{q}}_l \rfloor^{p_\mathcal{F}}\right] \right\| = 0.\end{aligned}$$

Using Property 3 and 4 we can shown that the second term in $\mathbf{f}_{NH3}$ also vanishes:

$$\begin{aligned}&\lim_{\epsilon \to 0} \frac{\left\| \mathbf{M}_l^{-1}(\epsilon^{r_1}\tilde{\mathbf{q}}_l + \mathbf{q}_*)\mathbf{C}_l(\epsilon^{r_1}\tilde{\mathbf{q}}_l + \mathbf{q}_*, \epsilon^{r_2}\dot{\mathbf{q}}_l)\epsilon^{r_2}\dot{\mathbf{q}}_l \right\|}{\epsilon^{2r_2 - r_1}} \\ &\leq \lim_{\epsilon \to 0} m_2 L_c \frac{\left\| \epsilon^{r_2}\dot{\mathbf{q}}_l \right\|^2}{\epsilon^{2r_2 - r_1}} = m_2 L_c \left\| \dot{\mathbf{q}}_l \right\|^2 \lim_{\epsilon \to 0} \epsilon^{r_1} = 0.\end{aligned}$$

Lastly, it only rest to prove that the non-homogeneous term $\mathbf{f}_{NH4}$ meets the *vanishing conditions*:

$$\begin{aligned}\mathbf{f}_{NH4} = &- \left[\mathbf{M}_r^{-1}(\tilde{\mathbf{q}}_r + \mathbf{q}_*) - \mathbf{M}_r^{-1}(\mathbf{q}_*)\right] \left[K_r \lceil \tilde{\mathbf{q}}_r - \tilde{\mathbf{q}}_l \rfloor^{p_\mathcal{U}} + B_r \lceil \dot{\mathbf{q}}_r \rfloor^{p_\mathcal{F}}\right] \\ &- \mathbf{M}_r^{-1}(\tilde{\mathbf{q}}_r + \mathbf{q}_*)\mathbf{C}_r(\tilde{\mathbf{q}}_r + \mathbf{q}_*, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r\end{aligned}$$

which proof follows the same steps as $\mathbf{f}_{NH3}$. With this, it only rest to invoke Lemma 2 to finish the proof of conclusion B and Proposition 1.

4 Experimental results

In this section, we present a comparison between the controller described in Proposition 1 against the well-known linear P+d controller with gravity cancellation [13]. For this, we used a six DoF teleoperation system composed of a local Kinova® Gen3 Ultra-lightweight and a remote Kinova® Gen3 Lite robotic arm. The experimental procedure consists in the following stages:

- First, both robots start at different positions, $q_l = [-20, 10, 20, 0, 20, 30]\frac{\pi}{180}$ for the local robot and $q_r = [0, 25, 35, 10, -10, 0]\frac{\pi}{180}$ for the remote.
- Then, the teleoperation system will try to minimize the position error between the robots, driving them to a consensus position.
- Finally, when the robots reach a common constant position, a human operator will apply external forces on the robots, showing the bounded behavior of the teleoperation system and the tracking capabilities of the controller.

Since the robots are composed of different motor sizes, it was necessary to set individual gains for each joint. The controller's gains selected for the experiments are: $K_r = [3, 17, 4, 1.5, 1.5, 1.5]$, $B_r = [0.5, 0.3, 0.3, 0.1, 0.2, 0.1]$, $K_l = [15, 15, 15, 15, 15, 15]$, $B_l = [2, 2, 2, 2, 2, 2]$, $r_1 = 1.5$ and $r_2 = 1$. For the linear scheme, we used the same gains K_l, K_r, B_l and B_r.

We started by testing the linear scheme. In Figure 1, we present the evolution of the joint position during the test for both manipulator. For the first 20 second of the experiment, it is observed how both robots tried to reach a consensus position, but ended in a steady state error. After 20 seconds, external forces were introduced in the system, showing lack of tracking capabilities for the linear scheme. Then, at 50 seconds the external forces ceased; however, the robots were not able to reach a common position.

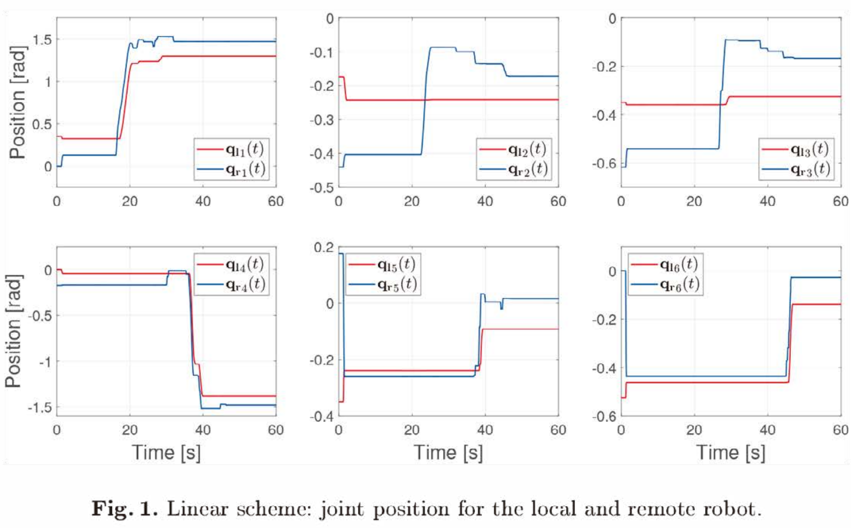

Fig. 1. Linear scheme: joint position for the local and remote robot.

In Figures 2, 3 and 4, we present a graph of the joint position of the robots and their corresponding velocities during the experiment that use the controller presented in Proposition 1. In the first 10 second, we see that both robots started at different positions and quickly reached a consensus position. After 10 seconds, we introduced external forces in every joint, where it can be seen that the controller was able to keep their positions synchronized with minimal error. Then, when the forces disappeared at around 40 seconds, both robots reach a consensus position again.

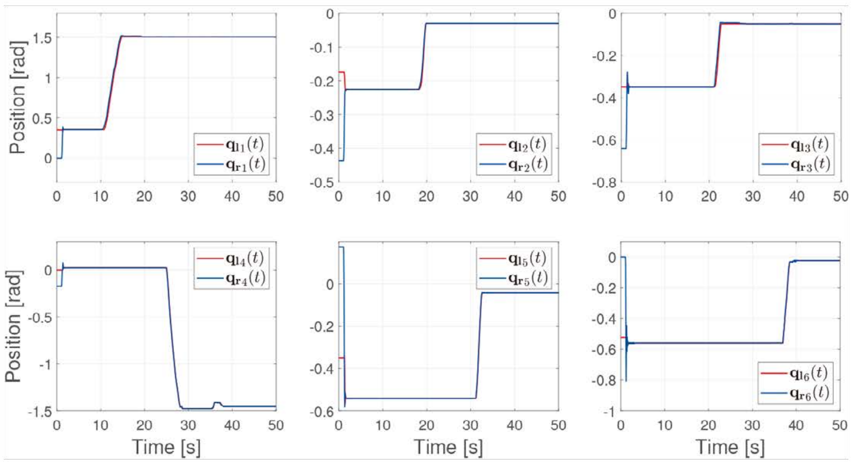

Fig. 2. Finite-time scheme: joint position for the local and remote robot.

5 Conclusions

This article proposed a continuous finite-time controller for bilateral teleoperation of fully actuated Euler-Lagrange systems. The control scheme, based on a proportional plus damping injection (P+d) approach with gravity compensation, ensures global finite-time convergence under the assumption of passive human-operator and remote-environment interactions. The theoretical analysis demonstrates that the position error and velocities remain bounded in the presence of external forces, while converging to zero in finite-time in their absence. Experimental results with a 6-DoF teleoperation system validate the effectiveness of the proposed controller, showcasing improved performance compared to a traditional linear P+d scheme. Future work will address practical challenges, such as the absence of velocity measurements and actuator force limitations.

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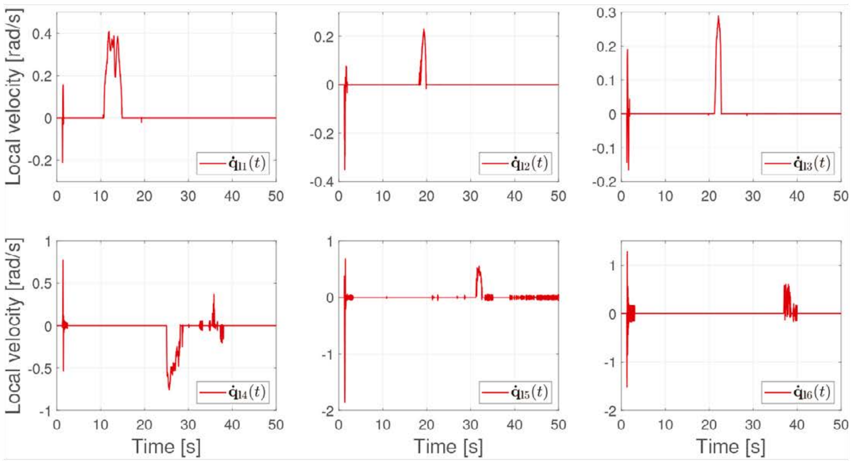

Fig. 3. Finite-time scheme: joint velocity for the local robot.

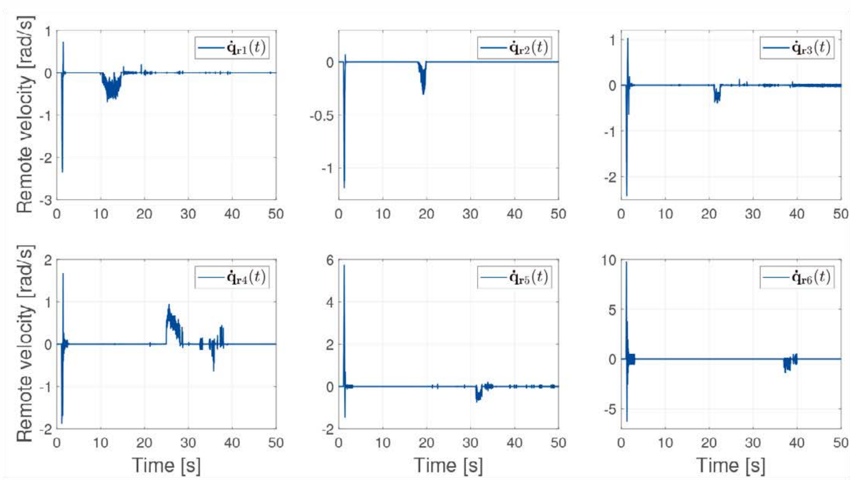

Fig. 4. Finite-time scheme: joint velocity for the remote robot.

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