Edge-Weighted Formation Control for Omnidirectional Mobile Robots

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Abstract. This paper presents a consensus-based formation control strategy for omnidirectional mobile robots based on weighted graphs. The proposed method drives a group of robots to achieve and maintain a desired formation, performing consensus in both positions and orientation agreements. Moreover, weighted graphs allow collision avoidance among robots by rotating and adapting the formation shape. Simulation experiments were performed to validate the effectiveness of the proposed strategy, demonstrating the successful convergence of formation in a network of four omnidirectional platforms.

Keywords: Formation control · Consensus · Omnidirectional Mobile Robots · Weighted Graphs · Multi-agent Systems.

1 Introduction

Formation control is a fundamental challenge in multi-agent systems, particularly in mobile robotics, where robots must coordinate their movements while maintaining a predefined geometric pattern. The significance of formation control is evident in various applications, including industrial automation, swarm robotics, search and rescue, and logistics [1]. Coordinated movement in multi-robot systems enhances efficiency, optimizes resource utilization, and enables robust performance in dynamic environments [2].

Traditional formation control approaches have included leader-follower structures, virtual structures, and behavior-based models, each offering unique advantages and trade-offs [3]. Leader-follower methods, for instance, rely on a designated leader dictating the movement of the group, which can introduce single-point failure risks. Virtual structure approaches treat the formation as a rigid body, ensuring precise control but reducing adaptability to obstacles. Behavior-based models, on the other hand, provide decentralized and flexible control but can suffer from instability in complex environments [4]. To overcome these limitations, consensus-based control strategies have emerged as an effective

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method of decentralized coordination [2]. These strategies enable robots to reach agreement on position through local interactions with neighboring agents.

Consensus control has been proven to be robust against external disturbances and communication constraints [5]. Furthermore, recent studies have introduced adaptive formation control mechanisms that integrate noise-tolerant consensus algorithms to improve formation robustness [4]. Such techniques have been particularly effective in real-world robotic applications, where unpredictable obstacles and communication delays can degrade control accuracy [1]. However, challenges remain when dealing with collision avoidance mechanisms to ensure safety and efficiency [3].

The consensus in both position and orientation agreement has been solved using persistent excitation term for a group of non-holonomic mobile robots [6,7]. However, omnidirectional mobile robots are free from non-holonomic constraints, then, it is not necessary to incorporate a persistent excitation, which can degrade overall performance.

A promising approach to solving these challenges is edge-weighted formation control, where inter-agent interactions are modeled as weighted connections [3,8]. Edge-weighted consensus formation control has gained traction due to its ability to dynamically adjust inter-robot influence. By assigning adaptive weights to edges in the communication graph, this approach allows for better stability, collision avoidance, and responsiveness to environmental changes. However, edge-weighted consensus strategy is applied to agreement only in position rather than orientations [9].

In this paper, we enable the edge-weighted consensus strategy to incorporate consensus in both position and orientation agreements for omnidirectional mobile robots. The effectiveness of the proposed method is validated through simulations conducted in CoppeliaSim based on four omnidirectional mobile robots.

The paper is organized as follows: The next section introduces the kinematics of omnidirectional mobile robots composed by four wheels. The section 3 address the edge-weighted consensus-based formation control. Then, simulations results are presented in section 4. Finally, the conclusions and future work are given in section 5.

2 Omnidirectional mobile robot kinematics

The omnidirectional mobile robot in this work is composed of four mecanum wheels, see Figure 1. The pose of the robot with respect to the world frame is given by the positions (x, y) and the orientation θ. The velocity v_m corresponds to the linear velocity of each wheel with $m = 1, 2, 3, 4$. The parameter L is half of the distance between the front and the rear wheels, and l is half of the distance between the left and right wheels.

The kinematics of the omnidirectional mobile platform is given by

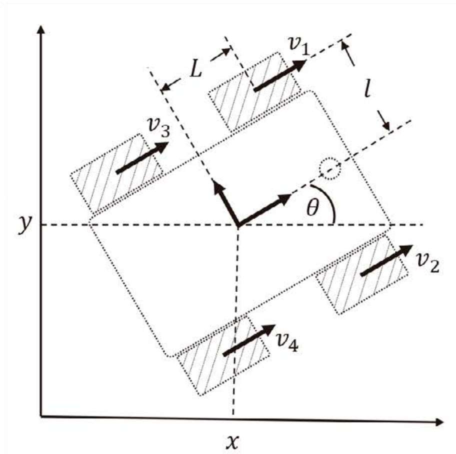


Fig. 1: Schematic of an omnidirectional mobile robot of four mecanum wheels.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} \sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) & -\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right) & -(L+l) \\ \sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right) & \sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) & (L+l) \\ \sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right) & \sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) & -(L+l) \\ \sqrt{2}\sin\left(\theta+\frac{\pi}{4}\right) & -\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right) & (L+l) \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \tag{1}$$

where $\dot{x}$ and $\dot{y}$ are the linear velocities and $\dot{\theta}$ is the angular velocity, all expressed in the world frame. These velocities will be designed for consensus formation control purposes. Detailed descriptions about kinematics can be found in [10,11].

3 Edge-Weighted Formation Control

The communication topology among a set of robots can be represented with an undirected graph with N elements defined as a pair $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set of cardinality M and $\mathcal{V} = \{\mathrm{v}_i, i = 1, 2, \cdots, N\}$ stands for the vertex set of cardinality N. An edge $\mathcal{E}_k = (\mathrm{v}_i, \mathrm{v}_j)$ represents bidirectional communication between agents i and j, where $\mathcal{E}_k$ is the k-th edge of G.

The position consensus problem for N agents can be solved with the Laplacian-based feedback method [3,9]. The feedback control is defined as

$$\dot{\mathbf{z}}_i = -\sum_{j \in \mathcal{N}_i} w_{ij}(\mathbf{z})(\mathbf{z}_i - \mathbf{z}_j), \tag{2}$$

where $\mathbf{z}_i := [x_i, y_i]^\top$, $\mathcal{N}_i = \{\forall\ \mathrm{v}_j \in \mathcal{V} \mid (i,j) \in \mathcal{E}\}$ is the Neighbors subset of the i-th robot. The edge-weight function $w_{ij}(\mathbf{z})$ is defined as

$$w_{ij}(\mathbf{z}) = \alpha_{ij}\left(-\frac{1}{k_{ij}}\text{csch}^2\left(\frac{\|\mathbf{z}_{ij}\| - \delta}{k_{ij}}\right) + 1\right), \tag{3}$$

where $\alpha_{ij} > 0$ defines the inter-robot position influence, $\mathbf{z}_{ij} := \mathbf{z}_i - \mathbf{z}_j$ is the position error, and $k_{ij} > 0$ is a constant related to the distance between robots. Indeed, parameter k_{ij} defines the final formation shape. Moerover, $\delta > 0$ is a safety distance parameter. Collision avoidance is guaranteed if δ is strictly greater than the distance $\|\mathbf{z}_{ij}\|$ [3].

In this work, we proposed to solve the orientation consensus problem as follows

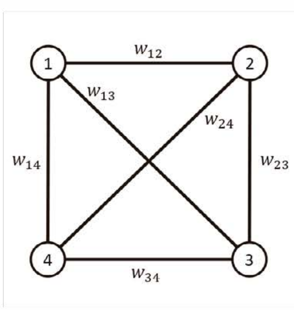
$$\dot{\theta}_i := -\sum_{j\in\mathcal{N}_i}\beta_{ij}(\theta_i - \theta_j), \tag{4}$$

where $\beta_{ij} > 0$ defines the inter-robot orientation influence.

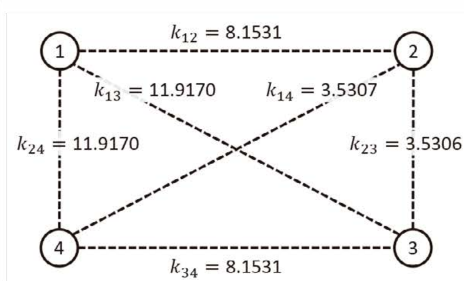
Finally, $\dot{\mathbf{z}}_i := [\dot{x}_i, \dot{y}_i]^\top$ and $\dot{\theta}_i$ of each agent i are mapped to omnidirectional wheel velocities using (1).

4 Simulation results

This section presents simulations to validate the proposed formation control in a group of four omnidirectional mobile robots. The communication topology and the desired formation pattern are presented in Fig. 2. The formation pattern is established by selecting appropriate k_{ij} values.



(a)



(b)

Fig. 2: Formation control topology. (a) Communication topology. (b) Desired formation.

The parameter configuration for the simulation are: $\alpha_{ij} = 0.25$, $\delta = 0.1m$, $\beta_{ij} = 0.8$, $L = 0.2355m$ and $l = 0.15m$. The controller parameters were experimentally selected. Moreover, control signals are bounded in the range $[-0.8, 0.8]$

to simulate actuator limitations. Finally, the proposed approach is tested in the Coppelia simulator. The simulator setup is given in Fig. 3.

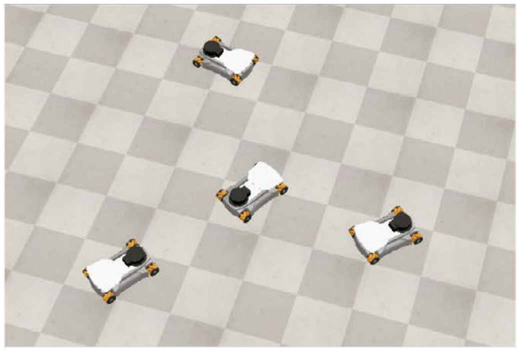


Fig. 3: Setup in Coppelia Simulator. Agents are shown in the initial configurations.

The position trajectories in the xy-plane and the resulting formation patterns are presented in Fig. 4. As can be seen, robots performed the desired formation pattern, see Fig. 2(b). The trajectories of agent 3 and agent 4 show that a possible collision is avoided. Notice that the formation shape is rotated, however, formation is achieved by avoiding collision among agents. Moreover, the final robot orientations are the same, which illustrates the consensus in orientations.

The time-varying position and orientation of agents are shown in Fig. 5. The Fig. 5(a) demonstrates smooth displacement to achieve consensus on positions after 6 seconds. Moreover, a consensus on orientation is reached after 3 seconds, see Figure 5 (b).

Control signals are illustrated in Fig. 6. At early seconds, signals are bounded, after that, the control signals are smooth and converge near zero after 10 seconds when the position and orientation consensus is reached.

5 Conclusions

This paper uses edge-weighted graphs to address the problem of formation control involving position and orientation consensus for omnidirectional mobile robots. The proposed method allows agents to achieve a desired formation pattern with smooth position control trajectories while also regulating the orientation of all robots to a common value. Additionally, position trajectories also demonstrate the capabilities of collision avoidance among agents. Moreover, Velocity control signals drive robots to consensus, even under velocity constraints, demonstrating the effectiveness of edge-weighted consensus control strategies.

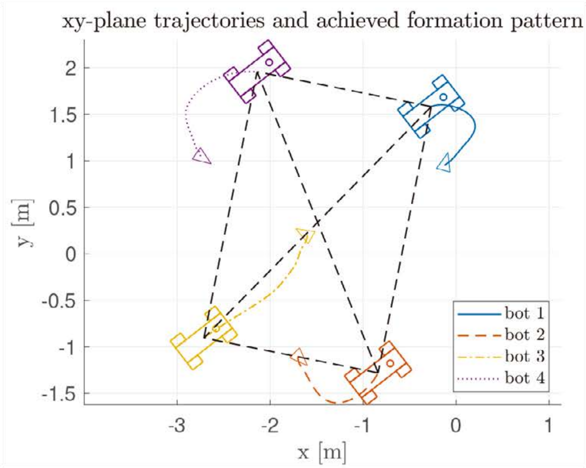
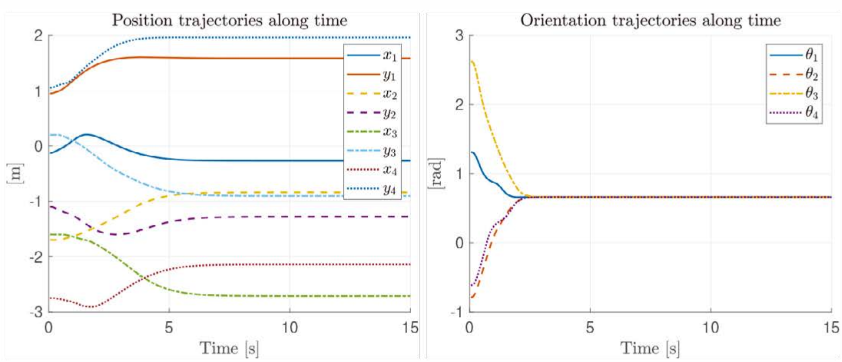


Fig. 4: Position trajectories and the achieved formation patterns. The initial position and orientation of each robot are represented by a triangle marker. The formation pattern achieved is highlighted with dotted lines.



(a) (b)

Fig. 5: Position (x_i, y_i) and orientation θ_i trajectories along time.

Implementing this approach in real-world experiments is left to future work. Further research may explore integrating machine learning-based adaptive weighting, refining the model for complex multi-robot formations, and investigating the impact of communication delays and network constraints on formation stability. These directions will contribute to advancing autonomous robotic coordination and cooperative multi-agent systems.

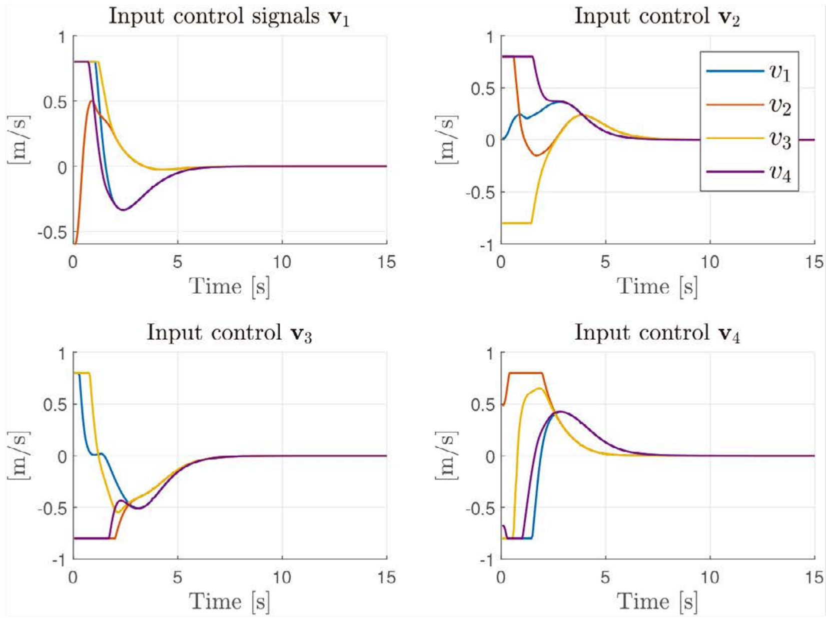


Fig. 6: Control input signals $\mathbf{v}_i := [v_1, v_2, v_3, v_4]^\top$ per along time per agent agent i with $i = 1, 2, 3, 4$.

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