# Identification of agricultural irrigation model via differential neural networks

Alma Y. Alanis Garcia and Ricardo R. Vázquez

Universidad de Guadalajara
alma.alanis@academicos.udg.mx
ricardo.reyes2801@alumnos.udg.mx
https://www.cucei.udg.mx/es/investigacion/cuerpos-academicos/sistemas-inteligentes-0

**Abstract.** Continuous monitoring of agricultural crops has traditionally been carried out through direct human supervision, either empirically or through instrumentation, this way in which agricultural workers monitor states of their production. This activity is very laborious since it involves constantly taking measurements and visual verifications during practically the entire evolutionary period within the crop to have certainty in the production yields for each established agricultural cycle.This article describes a method that uses a differential neural network that identifies the stages dynamic evolution; available water resource, amount used by plant and amount of material that fulfilled its life cycle, biomass, and that with accurate data it is possible to have better crop performance in each agricultural cycle as well as the care of the inputs and resources related to this important human activity.

**Keywords:** identification, differential neural network, evapotranspiration, algorithm, irrigation, agriculture

## 1 Introduction

Currently there are numerous strategies and ways to carry out control, identification, estimation and monitoring, all with applications to physical measurements of known signals, obtaining a permanent reading of the controlled variables, and through the execution of an algorithm or mathematical method. In this document it is based on the identification part that is fundamental for the success of a good control implementation and is carried out using a differential neural network [1][2][3]. Here this differential neural network DNN method is applied to a dynamic model of agricultural crop growth [9] which identifies parameters for their biological development through its variables, such as the amount of water needed and the amount of biomass developed [8].Constant monitoring of an agricultural crop traditionally carried out by a human here is assisted by DNN, a computational that integrates calculation and simulation tools, capable of identifying the system parameters in detail [11]. MATLAB®and Python are software that, among their multiple functionalities, contain modules that could

https://doi.org/10.61728/AE20255015

estimate the parameters of a plant[17][18]. It would also allow calculating its response to different input signals or physical variables to be measured, and thus determine the intelligent models that best fit the reality of a given system in the agricultural sector. Information on the status of crops is a growing demand from both farmers (to support precision farming methods) and institutions at different levels (to improve yield forecasts or ensure environmentally friendly practices)[12]. Agricultural management by growth stage is essential to optimize the yields of inputs such as nitrogen , plant growth regulator (PGR), fungicides and water [9].

## 2 Methodology

Identification is a method of collecting data to model the behavior of a given system [1][3]. In this paper, this identification incorporates a DNN differential neural network structure. These have the potential to model nonlinear systems that take into account the input/output relationship of a dynamic differential network. Therefore, with the purpose of carrying out the process of identifying the parameters of the system itself for state variables in the first instance for the agricultural sector, the dedicated structure to carry out this process is shown, Fig.1. Effectiveness has been recognized through the use of artificial neural networks for solving problems in reality that are based on an intelligent approach that is based on mathematical theorems that approximate for the identification of nonlinear systems [1] [13].

**Algorithm 1** algorithm DNN

1. $(x_n, y_n^*)$, Data Set $A, W_j$ initialize
2. for each $x \longleftarrow \hat{x}$
3. for each layer $\sigma_i(x) = a_i\left(1+e^{-b_i x_i}\right)^{-1} - c_i\phi_{ij}(x) = \bar{a}_{ij}\left(1+e^{-\sum_p \bar{b}_{ij}^p x_p}\right)^{-1} - \bar{c}_{ij}$
4. compute $(x_n, y_n^*)\mathrm{W}_j = -s_t[K_1 P \Delta_t \sigma(\hat{x}_t)^T]W_j = -s_t[K_2 P \Delta_t \gamma(u_t)^T \phi(\hat{x}_t)^T]$
5. for each weight $W_t \longleftarrow$ update
6. $\dot{V}_t \leq s_t[\Delta_t^T Q_0 \Delta_t + \bar{\eta}]$ is obtained $V_t$ bounded
7. $(x_n, y_n^*)$, update parameters $\theta$
8. while $\Delta_t, W_{1,t} + W_{2,t} \in\ L^\infty$ condition not satisfied
9. Repeat

where $W_j$ refers to the weights in node $i_j$.

Algorithm 1 shown in the pseudocode can be seen that learning law (12) of multilayer dynamic neural network (7) has a structure similar to the back propagation of multilayer perceptrons, Because it is derived using the Lyapunov approach, the stability of the global asymptotic error is guaranteed. Hence, the local minimum convergence problem.

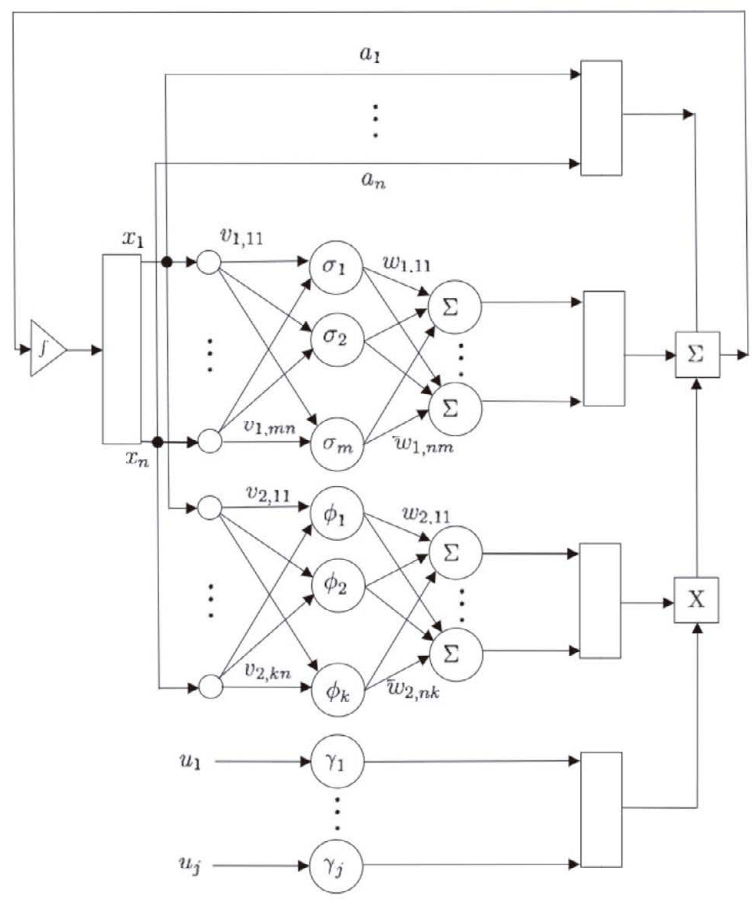


**Fig. 1.** neuro-dynamic network structure

## 3 Crop model formulation

It is well known that variations in water content affect plant growth, the water flow process forms a soil-plant system, where water variations directly affect the roots of the plant [7][8]. Description is represented by the following:

1. Soil water availability.
2. Water is available for growth.
3. dry bioamass number.

Process is the movement of molecules, water that enters is equal to that which is not used by transpiration and the stored :

$$\frac{dW}{dt} = \{Wabsorption\} - \{Wremoved\} \tag{1}$$

$$\{Wabsorption\} = \xi_4 \left( \frac{x_2}{1 + \xi_5} \right) r_t \tag{2}$$

$$\{Wremoved\} = \mu\rho(x)\xi_2 + x_2\xi_6 \tag{3}$$

in (3) the term $G(x)$ is considered by transpiration and $\mu$ is the plant growth rate

$$\dot{x}_3 = -\alpha_9 x_{3,t} - \frac{\sigma x_t}{1 + g x_t^{\psi}} \tag{4}$$

For plant growth [5], the variation in dry matter is considered, $x_t$ number of biomass processed in time , $\alpha_9$ correspond to the degradation rate of plant, $\sigma$ reason of essential intent and element $\psi$ drives speed growth, classification according to $\psi$:

- (a) the dry mass cannot grow; If $\psi < 0$, then $\rho(x) = \frac{\theta x}{1+g^{|-\psi|}}$, $x \gg 1$ tends $\theta x$.
- (b) $\psi = 0$, presents $\rho(x) = (\frac{\theta}{1+g})x$,biomass increases as area.
- (c) $\psi > 1$, $if x \gg 1$ then $\rho(x) = \frac{1}{x^{(\psi-1)}} \frac{\theta}{g}$ , this case is unusual since it quickly goes to zero.

below dynamic system, which represents the relationships given in equations (1) to (4).

$$\begin{cases} \dot{x}_1 = -\alpha_1 \dfrac{x_{1,t} x_{2,t}}{1 + \alpha_3 x_{2,t}} - \alpha_1 x_{1,t} + r_t \\ \dot{x}_2 = \alpha_4 \dfrac{x_{1,t} x_{2,t}}{1 + \alpha_3 x_{2,t}} - \alpha_5 x_{2,t} - \dfrac{\alpha_6 x_{3,t} x_{2,t}}{1 + \alpha_7 x_{3,t}} \\ \dot{x}_3 = -\alpha_9 x_{3,t} + \dfrac{\alpha_8 x_{2,t} x_{3,t}}{1 + \alpha_7 x_{3,t}} \end{cases} \tag{5}$$

Plant of the previous system (5) is defined $\Omega = (x_1, x_2, x_3) \in R^3 | \Omega \geq 0$.

Climatic parameters ultimately influence dynamic evolution of agricultural crops, solar irradiation, humidity, air, and also elemental inputs for their development.,as those described in Table 1. [9]

**Table 1.** Parameters description

| Parameters | description | units |
|---|---|---|
| $\alpha_1$ | *Reason* for pond water decrease. | $[\text{time}]^{-1}$ |
| $\alpha_2$ | *Water* consumed by plant | $[\text{time}\times mass]^{-1}$ |
| $\alpha_3$ | *Limiting* parameter of $x_{2,t}$ | $[\text{mass}^{-1}$ |
| $\alpha_4$ | *Intrinsic* increase of water within the plant | $[\text{time}\times mass]^{-1}$ |
| $\alpha_5$ | *Reason* for water decrease within the plant. | $[\text{time}]^{-1}$ |
| $\alpha_6$ | *Intrinsic* decrease in water by photosynthesis. | $[\text{time}\times mass]^{-1}$ |
| $\alpha_7$ | *Limiting* parameter of $x_{3,t}$ | $[\text{mass}]^{-1}$ |
| $\alpha_8$ | *Characteristic* ratio water within the plant. | $[\text{time}\times mass]^{-1}$ |
| $\alpha_9$ | *Plant* degradation | $[\text{time}]^{-1}$ |
| $x_{1,t}$ | *Water* available in the delimited area | $[\text{time}]^{-1}$ |
| $x_{2,t}$ | *Quantity* of internal water | $[\text{time}\times mass]^{-1}$ |
| $x_{3,t}$ | *Biomass* completed process | $[\text{time x mass}]^{-1}$ |
| $r_t$ | *Irrigation* action | $[\text{time x mass}]^{-1}$ |

A dynamic model system approach to agricultural systems markedly improves production in each cycle, which although this is outside the scope of this document, It has a great economic and labor impact environmental impact, in addition to the social components .

## 4 Differential neural network structure

Nonlinear system (5) and the DNN (7) are considered. If the assumptions are met [1][2][3]:

$$\begin{gathered} x_t = f(x_t, u_t, t) \\ x_t \in \Re^n, u_t \in \Re^m, n \geq m \end{gathered} \tag{6}$$

which analyzes following neuronal structure:

$$\begin{aligned} \frac{d\hat{x}}{dt} &= A\hat{x}_t + W_{1,t}\sigma(\hat{x}) + W_{2,t}\phi(\hat{x})u_t \\ &A \in R^{n\times n}, W_{1,t} \in R^{n\times n}, W_{2,t} \in R^{n\times n}, \sigma(\cdot) \in R^n \end{aligned} \tag{7}$$

and with sigmoidal activation functions that satisfy Lipschipz condition based on Lemma 1:

$$\begin{aligned}\sigma_i(x) &= a_i\left(1+e^{-b_i x_i}\right)^{-1} - c_i \\ \phi_{ij}(x) &= \bar{a}_{ij}\left(1+e^{-\sum_p \bar{b}_{ij}^p x_p}\right)^{-1} - \bar{c}_{ij}\end{aligned} \tag{8}$$

$$\begin{aligned}\|\sigma(x_t)-\sigma(x_t')\|^2_{\Lambda_\sigma} &\leq I_\sigma \|x_t - x_t'\|^2_{\Lambda_\sigma'} \\ \|\varphi(x_t)-\varphi(x_t')\|^2_{\Lambda_\varphi} &\leq I_\varphi \|x_t - x_t'\|^2_{\Lambda_\varphi},\end{aligned} \tag{9}$$

where $\Lambda_\varphi, \Lambda_\sigma$ are positive definite matrices.

Input signal is defined as:

$$r_t \leq \bar{r_t}$$

A strictly positive matrix $\Theta$ given that a matrix $A$ is stable, then the Riccati equation positive

$$A^T P + PA + P\Upsilon \mathrm{P} + \Theta = 0 \tag{10}$$

$$\begin{aligned}\Upsilon &= 2\overline{W}_1 + 2\overline{W}_2 + {\Lambda_f}^{-1} \\ \overline{W}_1 &:= W_1^{*T} {\Lambda_1}^{-1} W_1^* \\ \overline{W}_1 &:= W_2^{*T} {\Lambda_1}^{-1} W_2^* \\ \Theta &= \Theta_1 + \Lambda_\sigma + \bar{\mu}^2 \Lambda_\phi\end{aligned} \tag{11}$$

Weight updating rule contained in the differential neuronal structure shown in Fig.1 is shown.

$$\begin{aligned}W_1 &= -s_t[K_1 P \Delta_t \sigma(\hat{x}_t)^T] \\ W_2 &= -s_t[K_2 P \Delta_t \gamma(u_t)^T \phi(\hat{x}_t)^T]\end{aligned} \tag{12}$$

where dead zone is defined by:

$$s_t := 1 - \frac{\mu}{\|P^{\frac{1}{2}}\Delta\|} \tag{13}$$

$$\mu(t) = \frac{\bar{\eta}}{\lambda_{min}\frac{P}{Q_1 P^{-\frac{1}{2}}}}. \tag{14}$$

**Lemma 1.** *if differentiable vector given $\vartheta(x) \in \Re^m, (x \in \Re^m)$ satisfies the Lipschitz condition globally, presents the constant $I$*

$$\|f(x_1) - f(x_2)\| \leq I\|x_1 - x_2\|, I > 0 \tag{15}$$

*for any $x_1, x_2 \in \Re^n$, property for $x$, $\Delta x \in \Re^n$ :*

$$\|f(x+\Delta x=\vartheta(x)+\nabla^{T}g(x)\Delta x+v_{\vartheta} \tag{16}$$

*where the vector* $v_{\vartheta}$ *meets* $\|v_{\vartheta}\|\leq 2I_{g}\|\Delta x\|$

**Lemma 2.** *A positive function* $V(x)\in\Re^{n}$

$$V(x):=[\|x-x^{*}\|-\mu]_{+}^{2} \tag{17}$$

$[\bullet]_{x}^{2}:=([\bullet]_{+}^{2})[\bullet]_{+}$*defined :*

$$[\varsigma]_{+}=\begin{cases}\varsigma, & \varsigma\geq 0\\ 0, & \varsigma<0\end{cases} \tag{18}$$

*and V(x) is differentiable:*

$$\nabla V(x_{t}):=2[\|x_{t}-x_{t}^{*}\|-\varrho]_{+}\frac{x_{t}-x_{t}^{*}}{\|x_{t}-x_{t}^{*}\|} \tag{19}$$

*the LIpschitz constant = 1.*

**Theorem 1.** *For a system of equations form (6) in conjunction with neural network (7), it is true that:*

$$\Delta_{t},W_{1,t}+W_{2,t}\in\ I^{\infty} \tag{20}$$

*where identification error* $\Delta_{t}$ *meets following equation:*

$$\limsup_{t\to\infty}\frac{1}{T}\int_{t=0}^{T}\left(\ \Theta_{1}\Delta_{t}^{T}\Delta_{t}s_{t}dt\leq\bar{\eta}.\right. \tag{21}$$

*be candidate Lyapunov function and its derivative:*

$$V_{t}:=\Delta_{t}^{T}P\Delta_{t}+tr\left[\widetilde{W}_{1,t}^{T}K_{1}^{-1}\widetilde{W}_{1,t}\right]+tr\left[\widetilde{W}_{2,t}^{T}K_{2}^{-1}\widetilde{W}_{2,t}\right] \tag{22}$$

$$\begin{aligned}\dot{V}_{t}=&\frac{d}{dt}(\Delta_{t}^{T}P\Delta_{t})+\frac{d}{dt}tr\left[\widetilde{W}_{1,t}^{T}K_{1}^{-1}\widetilde{W}_{1,t}\right]\\&+\frac{d}{dt}tr\left[\widetilde{W}_{2,t}^{T}K_{2}^{-1}\widetilde{W}_{2,t}\right]\end{aligned} \tag{23}$$

*Given that* $s_{t}\geq 0$

$$\dot{V}_{t}\leq s_{t}\Delta_{t}^{T}I\Delta_{t}+I_{w1}+I_{w2}-\Delta_{t}^{T}\Theta_{1}s_{t}+\bar{\eta}s_{t}$$

(24)

where:

$$I = A^T P + PA + P\Upsilon P + \Theta$$

$$2trI_{w1} = \left[\dot{\widetilde{W}}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] + 2\Delta_t^T P \widetilde{W}_{1,t} \sigma s_t$$

$$2trI_{w1} = \left[\dot{\widetilde{W}}_{1,t}^T K_1^{-1} \widetilde{W}_{1,t}\right] + 2\Delta_t^T P \widetilde{W}_{1,t} \mu s_t$$

equating learning matrices:

$$\dot{\widetilde{W}}_{1,t} = \dot{W}_{1,t}, \dot{\widetilde{W}}_{2,t} = \dot{W}_{2,t} \tag{25}$$

is obtained

$$I_{w1} = 0, I_{w2} = 0 \tag{26}$$

is obtained $V_t$ bounded

$$\dot{V}_t \leq s_t[\Delta_t^T \Theta_0 \Delta_t + \bar{\eta}] \tag{27}$$

Right side of this inequality can be estimated as follows:

$$\dot{V}_t \leq s_t \lambda_{min} \left[P^{\frac{1}{2}} \Theta_1 P^{-\frac{1}{2}}\right] \left[\|P^{\frac{1}{2}} \Delta\|^2 - \mu\right] \leq 0 \tag{28}$$

Since $0 \leq st \leq 1$, produces:

$$\dot{V}_t \leq -\Delta_t^T \Theta_0 \Delta_t \bar{\eta} s_t \leq -\Delta_t^T \Theta_0 \Delta_t s_t + \bar{\eta} \tag{29}$$

Integrating (27) from 0 to $T$ :

$$V_T - V_0 \leq -\int_{t=0}^{T} ( \Theta_1 \Delta_t^T \Delta_t s_t dt \leq \bar{\eta} \tag{30}$$

Since $W_{1,0} = W_{1,0}^* \text{and} W_{2,0} = W_{2,0}^*, V_0$ are bounded

$$\int_{t=0}^{T} ( \Theta_1 \Delta_t^T \Delta_t s_t dt \leq V_0 - V_T \bar{\eta} \leq V_0 + \bar{\eta} T \tag{31}$$

## 5 Simulation results

DNN constants initialization:

$$A_0 = \begin{pmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{pmatrix} \hat{x}_0 = \begin{pmatrix} 7.5 & 0 & 0 \end{pmatrix}$$

$$W_1 = W_2 = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ 7 & -4 & 6 \end{pmatrix} P = \begin{pmatrix} 0.15 & 0.08 & -0.1 \\ 0.08 & 0.32 & -0.22 \\ -0.1 & 0.22 & 0.36 \end{pmatrix}$$

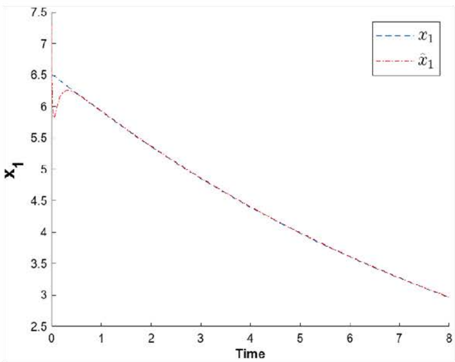


**Fig. 2.** First state of identifier converging true system state; $\alpha_2 = 2.0(time \times mass)^{-1}, \alpha_8 = 0.01(time \times mass)^{-1}, \alpha_9 = 0.0001(time)^{-1}$

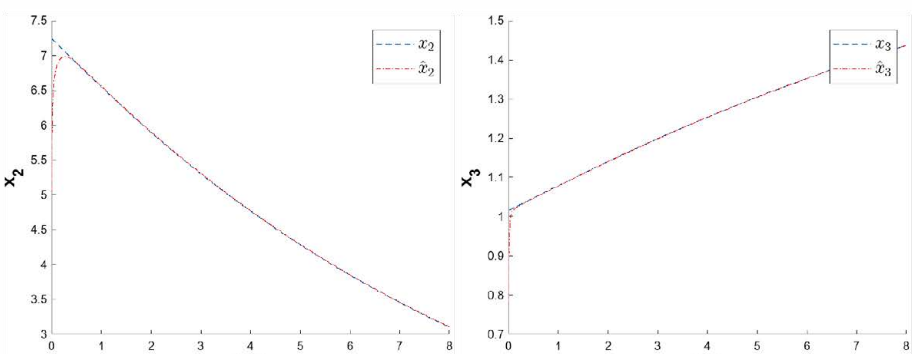


**Fig. 3.** Second and third states of identifier converging true system state; $\alpha_1 = 0.0001(time)^{-1}, \alpha_3 = 20(mass)^{-1}, \alpha_4 = 0.1(time \times mass)^{-1}, \alpha_5 = 0.1(time)^{-1}, \alpha_6 = 0.1(time \times mass)^{-1}, \alpha_7 = 0.1(mass)^{-1}$

Identification simulation shows characteristics achieved over time by dynamic system (5), which are shown by means of DNN initialization constants matrices A, $W_1, W_2$; first in Fig. 2 the convergence of state $x_1$ towards the natural evolution of the trajectory (available water) is observed with $\alpha_2$,$\alpha_8$ and $\alpha_9$ parameters arranged below this same graph, second in Fig. 3 dynamic evolutionary trajectories of states $x_2$ and $x_3$ are shown which in conjunction with the $\alpha_1$,$\alpha_3$,$\alpha_4$,$\alpha_5$,$\alpha_6$ and $\alpha_7$ parameters show the temporary consumption of water by the plant and reach of maturity of its biomass. Evolution of states are captured in these simulations which in applications of DNN proposal in this document give a very approximate graphic identification result as those shown above.

## 6 Conclusion

DNN is applied to identification dynamic agriculture growth (evapotraspiration).Through it, a numerical example was developed to show how proposed identifier will be used to approximate the dynamics of the system of equations and its states are shown in graphic form.Mathematical use form especific model result useful applied agriculture area with model every time better aproximated by hand of artificial neurons witch in document is applied throuuh a specific structure that provides a high quality result and a well-supported numerical interpretation all this in company of computational numerical software.

## References

1. Edgar N Sanchez,Alexander S Poznyak, “Neural Networks for Robust Nonlinear Control Identification”, State Estimation and Trajectory Tracking ,London, U.K. World Scientific ,https://doi.org/10.1142/4703

2. A. S. Poznyak, Wen Yu, E. N. Sanchez and J. P. Perez, ”Nonlinear adaptive trajectory tracking using dynamic neural networks,” in IEEE Transactions on Neural Networks, vol. 10, no. 6, pp. 1402-1411, Nov. 1999, doi: 10.1109/72.809085.

3. I. Nachevsky, O. Andrianova, I. Chairez and A. Poznyak, “Differential Neural Network Identifier for Dynamical Systems With Time-Varying State Constraints,” in IEEE Transactions on Neural Networks and Learning Systems, vol. 36, no. 1, pp. 407-418, Jan. 2025, doi: 10.1109/TNNLS.2023.3326450.

4. Edgar N. Sanchez, Alexander S. Poznyak,”Nonlinear Adaptive Trajectory Tracking Using Dynamic Neural Networks” 2001 World Scientific Publishinghttps://doi.org/10.1142/4703

5. Allen, R.G.; Pereira, L.S.; Raes, D.; Smith, M. “Crop Evapotranspiration-Guidelines for Computing Crop Water Requirements-FAO Irrigation and Drainage” Paper 56; FAO: Rome, Italy, 1998; Volume 300, p. D05109.

6. Márcia Lemos-Silva, Sandra Vaz, Delfim F. M. Torres, "Stability analysis and optimal control of the logistic equation" arXiv:2409.09077v1[math.OC] 9 Sep 2024, https://doi.org/10.48550/arXiv.2409.09077

7. Waller, P. Yitayew, M. "Irrigation and Drainage Engineering" Springer: Berlin/Heidelberg, Germany, 2015 DOI: 10.1007/978-3-319-05699-9

8. Tremblay, M.; Wallach, D. "Comparison of parameter estimation methods for crop models" Agronomie 2004, 24, 351–365 DOI: 10.1051/agro:2004033

9. Lopez-Jimenez, J.; Vande Wouwer, A.; Quijano, N. "Dynamic Modeling of Crop–Soil Systems to Design Monitoring and Automatic Irrigation Processes: A Review with Worked Examples" Water 2022, 14, 889. https://doi.org/10.3390/w14060889

10. Chrouta, J.; Chakchouk, W.; Zaafouri, A.; Jemli, M. "Modeling and control of an irrigation station process using heterogeneous cuckoo search algorithm and fuzzy logic controller" IEEE Trans. Ind. Appl. 2018, 55, 976–990.

11. Fourcaud, T. Zhang, X. Stokes, A. Lambers, H. Körner, C. "Plant growth modelling and applications: The increasing importance of plant architecture in growth models" Ann. Bot. 2008, 101, 1053–1063 DOI: 10.1109/TIA.2018.2871392

12. Capraro, F. Tosetti, S. Rossomando, F. Mut, V.; Vita Serman, F. "Web-based system for the remote monitoring and management of precision irrigation: A case study in an arid region of Argentina" Sensors 2018, 18, 3847. DOI: 10.3390/s18113847

13. Khalil, H.K. "Nonlinear Systems" ,Prentice-Hall, Inc.: Hoboken, NJ, USA,1996; pp. 102–103.

14. Rosegrant, M.W.; Ringler, C.; Zhu, T. "Water for agriculture: Maintaining food security under growing scarcity", Annu. Rev. Environ. Resour. 2009, 34, 205–222. DOI: 10.1146/annurev.environ.030308.090351

15. Zare, E. Arshad, M. Zhao, D. Nachimuthu, G. Triantafilis, J. "Two-dimensional time-lapse imaging of soil wetting and drying cycle using EM38 data across a flood irrigation cotton field", Agric. Water Manag. 2020, 241, 106383. DOI: 10.1016/j.agwat.2020.106383

16. Keesman, K.J. "System Identification: An Introduction", Springer Science Business Media: Berlin/Heidelberg, Germany, 2011

17. Jinkun Liu, "Intelligent Control Design and MATLAB Simulation" , Spriengr, 2018

18. Teik Toe Teoh , Zheng Rong, "Artificial Intelligence with Python,Machine Learning: Foundations, Methodologies, and Applications", Spriengr, 2022 https://doi.org/10.1007/978-981-16-8615-3